Physica A 390 (2011) 1350–1355

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Analysis of dynamics in magnetotelluric data by using the Fisher–Shannon method

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ARTICLE INFO

Article history:
Received 9 September 2010
Received in revised form 5 November 2010
Available online 22 December 2010

Keywords:
Magnetotellurics
Fisher information measure
Shannon entropy

ABSTRACT

The Fisher–Shannon information (FS) plane, defined by the Fisher information measure and the Shannon entropy power, is used to investigate the complex dynamics of magnetotelluric data of three stations in Taiwan. In the FS plane the electric and magnetic components are significantly separated, characterized by different degrees of order. Further investigation shows that signals measured in areas with very high level of seismic activity are well discriminated.

1. Introduction

Natural electromagnetic (EM) field on the Earth's sky is changed due to the solar wind and lightning activities and the secondary induction EM wave is propagated in the solid Earth. The magnetotelluric (MT) method is a technique measuring the variation in the EM field on the Earth's surface for geological mapping of the subsurface structures [1]. The MT data include two electric (Ex, Ey) and two magnetic (Hx, Hy) components, both in horizontal layouts with the north–south and east–west directions, and one vertical magnetic (Hz) component. The fundamental MT transfer function between the amplitudes of electric and magnetic fields can be written as $\mathbf{E}i(\omega) = \mathbf{Z}ij(\omega) * \mathbf{H}j(\omega)$, where ω is the frequency and $\mathbf{Z}$ is the impedance tensor related to the electrical properties of subsurface materials.

In this study, we analyzed the observed five channels MT time series. In order to study the dynamics of these MT data, we used the Fisher information measure and the Shannon entropy. We clearly find that the dynamical behavior for the electrical and magnetic fields is very different in our seven days data.

2. Data description

The MT data were continuously recorded with 15 Hz sampling rate by the MTU-5A systems (Phoenix Ltd., Canada) at three sites around the Taiwan area (Fig. 1). The recording span at each site is one week; the data was recorded at Sites 1638 and 1878 from 5 September 2009 through 12 September 2009 and at Site 1871 from 20 through 27 July 2009. These three sites are located in different seismicity zones (Fig. 1).

The MT data were first re-sampled from 15 Hz down to 1 min, leading to the datasets of 1440 points for one channel everyday. We thus investigated the dynamics of the daily records of the minute means of the two electric and three magnetic

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doi:10.1016/j.physa.2010.12.005

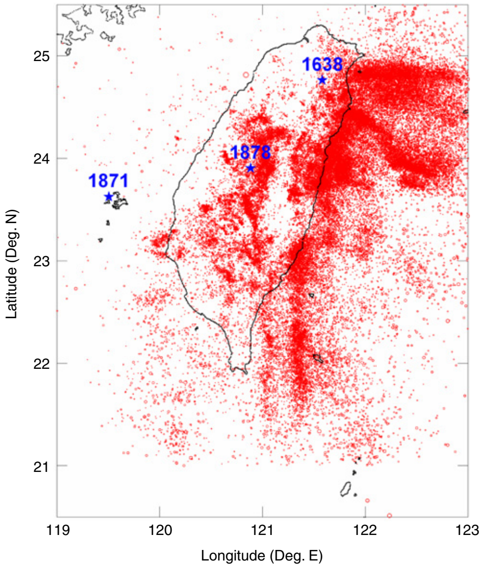

Fig. 1. Locations of MT observation sites (blue stars) with regional earthquakes ad Taiwan, drawn in red circles ($M > 3$, from 1973 to 2004).

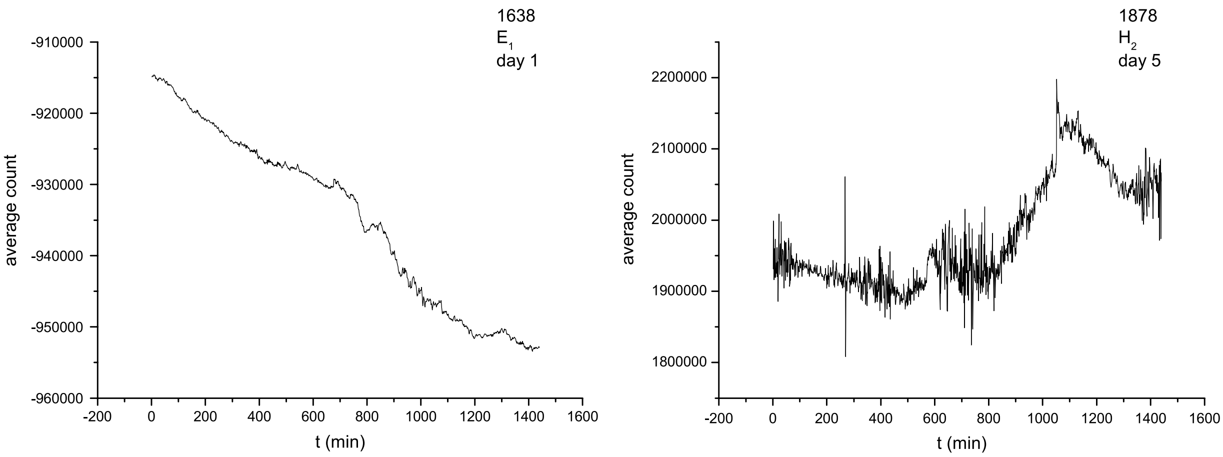

Fig. 2. Examples of a daily record of electric and magnetic components. The electric and magnetic signals are given as counts, which can be converted as mV/km and nT respectively by means of appropriate conversion formulae.

components at the three magnetotelluric stations. In order to remove the nonstationarities of the first order, at least, we analyzed the first difference time series. Fig. 2 shows as an example, the daily record of electric and magnetic components in two different stations. Fig. 3 shows the first difference time series of the data shown in Fig. 2.

3. Fisher information analysis and Shannon entropy

The Fisher Information Measure (FIM) is a powerful tool to investigate complex and nonstationary signals; the Shannon entropy is the well-known magnitude to quantify the degree of disorder in dynamical systems. The FIM was introduced by Fisher in 1925 in the context of statistical estimation [2]. In a seminal paper Frieden has shown that FIM is a versatile tool to describe the evolution laws of physical systems [3]. FIM allows to accurately describe the behavior of dynamic systems, and to characterize the complex signals generated by these systems [4]. This approach has been used by Martin et al. to characterize the dynamics of EEG signals [5]. Martin et al. have shown the informative content of FIM in detecting significant changes in the behavior of nonlinear dynamical systems [6] disclosing, thus, FIM as an important quantity involved in many aspects of the theoretical and observational description of natural phenomena. The FIM was used in studying several geophysical and environmental phenomena, revealing its ability in describing the complexity of a system [7–9] and suggesting its use as to reveal reliable precursors of critical events [10–13].

The Shannon entropy is a measure of the amount of information in a certain information source and represents the degree of indeterminacy in a certain system [14]. The Shannon entropy can be used to define the degree of uncertainty

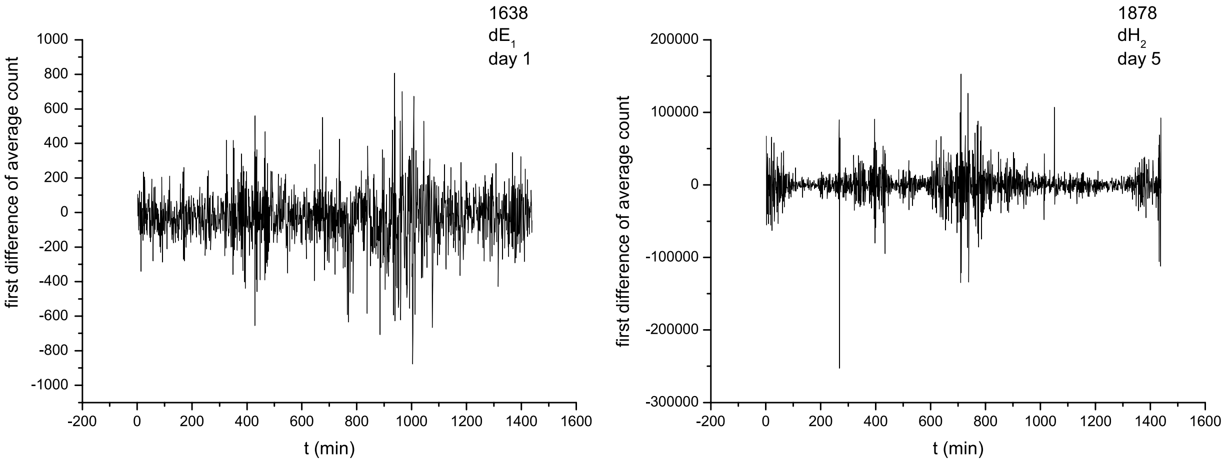

Fig. 3. First difference of the time series shown in Fig. 2.

involved in predicting the output of a probabilistic event [15]. For discrete distributions, this means that if one predicts the outcome exactly before it happens, the probability will be a maximum value and, as a result, the Shannon entropy will be a minimum. If one is absolutely able to predict the outcome of an event, the Shannon entropy will be zero. Such is not the case for distributions (probability densities) on a continuous variable, ranging e.g. over the real line. In this case, the Shannon entropy can reach any arbitrary value, positive or negative. Therefore, the use of the power entropy (that is defined below) avoids the difficulty of dealing with negative information measures. Shannon entropy provides a scientific method to understand the essential state of things [16,17].

Let us introduce the relevant Fisher- and Shannon-associated quantities [6]. Let $f \equiv q^2$ be a probability density in $\Re^d$ $(d \geq 1)$. Fisher's quantity of information associated to f (or to the probability amplitude q) is defined as the (possibly infinite) non-negative number I

$$I(f) = \int_{\Re^d} \mathrm{d}\mathbf{x} \frac{|\nabla f|^2}{f} \tag{1}$$

or in terms of the amplitudes

$$I(q) = \int_{\Re^d} \mathrm{d}\mathbf{x} (\nabla q \cdot \nabla q) \tag{2}$$

where ∇ is the differential operator. This formula defines a convex, isotropic functional I, which was first used by Fisher [2] for statistical purposes. It is clear from Eq. (2) that the integrand, being the scalar product of two vectors, is independent of the reference frame [6].

Let us focus the attention on the one-dimensional case. Let us consider a measurement x whose probability density function is denoted as $f(x)$. Its FIM is defined as

$$I = \int_{-\infty}^{+\infty} \left(\frac{\partial}{\partial x} f(x) \right)^2 \frac{\mathrm{d}x}{f(x)}. \tag{3}$$

The Shannon entropy is given by the following formula [4]:

$$H_X = -\int_{-\infty}^{+\infty} f_X(x) \log f_X(x) \, \mathrm{d}x. \tag{4}$$

For convenience the alternative notion of entropy power [18]

$$N_X = \frac{1}{2\pi e} \mathrm{e}^{2H_X} \tag{5}$$

will be used rather than the entropy H_X. The use of the power entropy N_X instead of the Shannon one H_X arises from the so-called 'isoperimetric inequality' [18–21], a lower bound to the Fisher–Shannon product which reads as $IN_X \geq d$, where d is the dimension of the space. The 'isoperimetric inequality' suggests that the FIM and the Shannon entropy are intrinsically linked, so that the dynamical characterization of signals should be improved when analyzing them in the so called Fisher–Shannon (FS) information plane [4], in which the y- and x-axis are the FIM and the Shannon entropy (as outlined above, instead of the Shannon entropy we will use the entropy power N_X). Vignat and Bercher [4] showed that the simultaneous examination of both Shannon entropy and FIM through the FS plane could improve the characterization of the non-stationary behavior of complex signals, like the Tsallis and the power exponential signals. The product IN_X can be considered

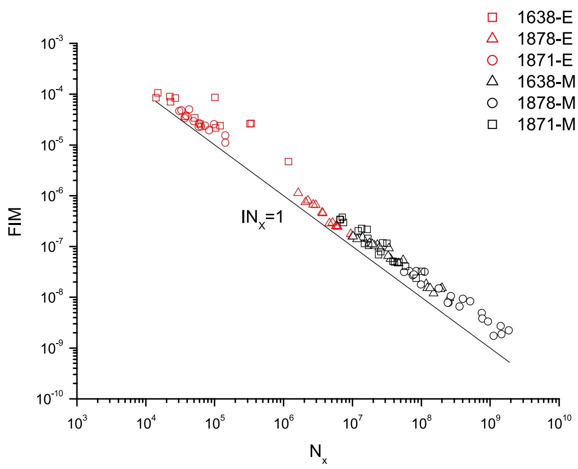

Fig. 4. FS plane for the magnetotelluric data measured in the three sites in Taiwan. The black symbols correspond to the magnetic component and the red ones to the electric component. The 'isocomplexity line' $IN_X = 1$ is also represented and separates the FS plane into two regions: one allowed and one not-allowed. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

as a statistical measure of complexity [20]. The line $IN_X = 1$ separates the FS plane in two parts: one allowed ($IN_X > 1$) and one not allowed ($IN_X < 1$), and the distance of a signal point from the 'isocomplexity line' $IN_X = 1$ can measure the degree of complexity of the signal.

Eqs. (3) and (4) involve the calculation of the probability density function (pdf) $f(x)$.

An estimation of the pdf $f(x)$ may be obtained by means of the kernel density estimator technique [22,23]. The kernel density estimator provides an approximate value of the density in the form

$$\hat{f}_M(x) = \frac{1}{Mb}\sum_{i=1}^{M} K\left(\frac{x-x_i}{b}\right) \tag{6}$$

where M is the number of data and $K(u)$ is the kernel function, which is a continuous non-negative and symmetric function satisfying

$$K(u) \geq 0 \quad \text{and} \quad \int_{-\infty}^{+\infty} K(u)\mathrm{d}u = 1, \tag{7}$$

whereas b is the bandwidth. In our estimation procedure the kernel used is the Gaussian of zero mean and unit variance. In this case

$$\hat{f}_M(x) = \frac{1}{M\sqrt{2\pi b^2}}\sum_{i=1}^{M} \mathrm{e}^{-\frac{(x-x_i)^2}{2b^2}}. \tag{8}$$

The Gaussian kernel allows to evaluate the kernel density estimator and the bandwidth with a low computational complexity [24].

4. Results and discussion

The analysis of the magnetotelluric data was performed through the following steps: (i) collecting a discrete and finite set of MT data, (ii) estimating the sequence by means of a continuous probability distribution function (pdf) by using the kernel density estimator technique, and (iii) quantifying the Fisher information and the Shannon entropy of the continuous distribution, in order to build up the information plane associated to the recorded data.

Fig. 4 shows the Fisher–Shannon information plane for magnetotelluric data measured in the three stations in Taiwan. Each symbol represents a daily record of the electric or magnetic component at one station. It is clearly visible that the electric components occupy the region of the FS plane characterized by lower Shannon entropy and higher FIM, while the magnetic components are mainly characterized by higher Shannon entropy and lower FIM. It is visible a kind of aggregation of the two components in two quasi-separate ensembles. Each ensemble, given by the points corresponding to the FS values for each signal is well discriminated from the other. From these results it can be deduced that the content of information or the degree of order is higher in the electric than in the magnetic component.

Fig. 5 shows the FS values only for the electric component. It is visible a clear separation between the signals measured at stations 1638 and 1871 from those measured at the station 1878, which is located in the most seismically active area of Taiwan after the 1999 Mw 7.6 Chi-Chi earthquake. In particular the FIM values for the electric components of 1878 channel

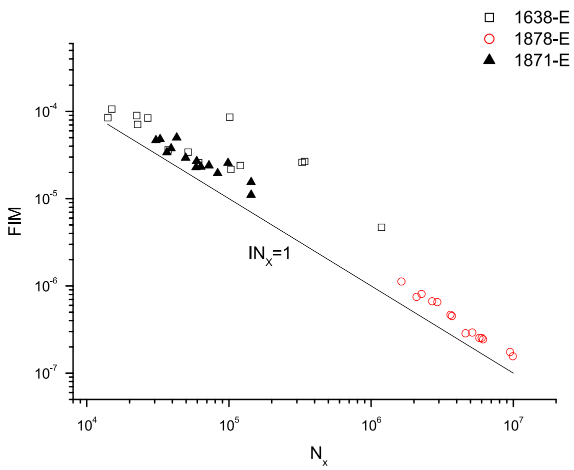

Fig. 5. FS plane for the electric component measured in the three sites in Taiwan.

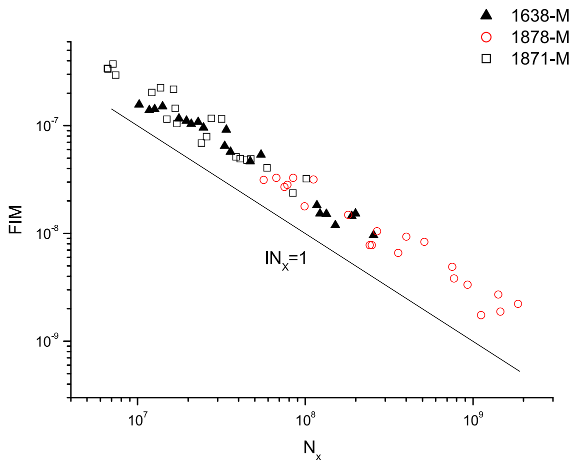

Fig. 6. FS plane for the magnetic component measured in the three sites in Taiwan.

are relatively very low, and, correspondingly the values of the Shannon entropy are relatively very high. Fig. 6 shows similar results but with regard to the magnetic component; also in this case, the discrimination between the FS values of the 1871 and 1638 signals from those of the 1878 signals is rather good. These results show that in very seismically active areas of Taiwan the degree of order in magnetotelluric data is relatively low.

Although the geophysical interpretation of such results is not a simple task, it is noteworthy that the FS information plane seems to be a good tool for discriminating signals measured in areas with very different level of seismic activity. The capability of MT method to investigate the electrical properties of the Earth's crust at large sounding depths, in particular those comparable with hypocentral depths, is well-known; therefore it is reasonable that the time dynamics of MT signals, measured in areas with a so high level of seismic activity, should be influenced by such enhanced activity, which breaks the order of the dynamics of the generated signals. Balasco et al. [8] analyzing the FIM of the magnetotelluric Earth's apparent resistivity found that the apparent resistivity is governed by more irregular dynamics (low FIM) at large sounding depths (then comparable with hypocentral depths), for which only random fluctuation are present on the impedance estimates. Therefore, the results of the present paper seem to be consistent with such finding.

5. Conclusions

In the present paper, we analyzed the magnetotelluric signals measured in three sites in Taiwan by means of the the Fisher–Shannon information plane, defined by the Fisher information measure and the Shannon entropy power. Our findings point out to a discrimination in the FS plane of the electric and magnetic components and of signals measured in a very seismically active areas from those measured in areas with lower level of tectonic activity. Our results, even if still preliminary, suggest that the FS information plane could be a good tool to better investigate the complex dynamics of magnetotelluric signals and, perhaps, reveal a new way for identifying the potential areas of earthquake hazard.

Acknowledgements

L.T. acknowledges the financial support of CNR in the frame of the CNR-NSC Bilateral Agreement for Scientific and Technological Cooperation. C.C.C. is grateful for the research support from both the National Science Council (ROC) and the Institute of Geophysics (NCU, ROC).

References

[1] C.C. Chen, S.C. Chi, C.S. Chen, C.H. Yang, Electrical structures of the source area of the 1999 Chi-Chi, Taiwan, earthquake: spatial correlation between crustal conductors and aftershocks, Tectonophysics 443 (2007) 280–288.
[2] R.A. Fisher, Theory of statistical estimation, Proc. Camb. Phil. Soc. 22 (1925) 700–725.
[3] B.R. Frieden, Fisher information, disorder, and the equilibrium distributions of physics, Phys. Rev. A 41 (1990) 4265–4276.
[4] C. Vignat, J.-F. Bercher, Analysis of signals in the Fisher–Shannon information plane, Phys. Lett. A 312 (2003) 27–33.
[5] M.T. Martin, F. Pennini, A. Plastino, Fisher's information and the analysis of complex signals, Phys. Lett. A 256 (1999) 173–180.
[6] M.T. Martin, J. Perez, A. Plastino, Fisher information and nonlinear dynamics, Physica A 291 (2001) 523–532.
[7] L. Telesca, R. Caggiano, V. Lapenna, M. Lovallo, S. Trippetta, M. Macchiato, The Fisher information measure and Shannon entropy for particulate matter measurements, Physica A 387 (2008) 4387–4392.
[8] M. Balasco, V. Lapenna, M. Lovallo, G. Romano, A. Siniscalchi, L. Telesca, Fisher information measure analysis of earth's apparent resistivity, Int. J. Nonlinear Sci. 5 (2008) 230–236.
[9] L. Telesca, R. Caggiano, V. Lapenna, M. Lovallo, S. Trippetta, M. Macchiato, Analysis of dynamics in Cd, Fe, and Pb in particulate matter by using the Fisher–Shannon method, Water Air Soil Pollut. 201 (2009) 33–41.
[10] L. Telesca, M. Lovallo, A. Ramirez-Rojas, F. Angulo-Brown, A nonlinear strategy to reveal seismic precursory signatures in earthquake-related self-potential signals, Physica A 388 (2009) 2036–2040.
[11] L. Telesca, V. Lapenna, M. Lovallo, Fisher information analysis of earthquake-related geoelectrical signals, Nat. Hazards Earth Syst. Sci. 5 (2005) 561–564.
[12] L. Telesca, V. Lapenna, M. Lovallo, Fisher information measure of geoelectrical signals, Physica A 351 (2005) 637–644.
[13] L. Telesca, M. Lovallo, R. Carniel, Time-dependent Fisher information measure of volcanic tremor before 5 April 2003 paroxysm at Stromboli volcano, Italy. J. Volcanol. Geoter. Res. 195 (2010) 78–82.
[14] C.E. Shannon, A mathematical theory of communication, Bell Syst. Tech. J. 27 (1948) 379–423, 623–656.
[15] R.C. Hilborn, Chaos and Nonlinear Dynamics, Oxford University Press, Oxford, 1994.
[16] S. Fuhrman, M.J. Cunningham, X. Wen, G. Zweiger, J.J. Seilhamer, R. Somogyi, The application of Shannon entropy in the identification of putative drug targets, Biosystems 55 (2000) 5–14.
[17] D.B. de Araujo, W. Tedeschi, A.C. Santos, J. Elias Jr., U.P.C. Neves, O. Baffa, Shannon entropy applied to the analysis of event-related fMRI time series, NeuroImage 20 (2003) 311–317.
[18] A. Dembo, T.A. Cover, J.A. Thomas, Information theoretic inequalities, IEEE Trans. Inform. Theory 37 (1991) 1501.
[19] E. Romera, J.S. Dehesa, The Fisher–Shannon information plane, an electron correlation tool, J. Chem. Phys. 120 (2004) 8906–8912.
[20] J.C. Angulo, J. Antolin, K.D. Sen, Fisher–Shannon plane and statistical complexity of atoms, Phys. Lett. A 372 (2008) 670–674.
[21] R.O. Esquivel, J.C. Angulo, J. Antolin, J.S. Dehesa, S. Lopez-Rosa, N. Flores-Gallegos, Analysis of complexity measures and information planes of selected molecules in position and momentum spaces, Phys. Chem. Chem. Phys. 12 (2010) 7108–7116.
[22] L. Devroye, A Course on Density Estimation, Birkhauser, Boston, 1987.
[23] A. Janicki, A. Weron, Simulation and Chaotic Behavior of Stable Stochastic Processes, Marcel Dekker, New York, 1994.
[24] V.C. Raykar, R. Duraiswami, Fast optimal bandwidth selection for kernel density estimation, in: Proceedings of the sixth SIAM International Conference on Data Mining, Bethesda, April 2006, pp. 524–528.