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Precursory phenomena associated with large avalanches in the long-range connective sandpile model II: An implication to the relation between the *b*-value and the Hurst exponent in seismicity

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[1] We analyze the Hurst exponent *H* and a power-law exponent *B* obtained from frequency-size distributions of avalanche events in the long-range connective sandpile (LRCS) model and study the relation between those two exponents. The LRCS model is introduced by considering the random distant connection between two separated cells. We find that the *B*-values typically reduce prior to large avalanches while the *H*-values increase. Both parameters appear precursory phenomena prior to large avalanche events. Most importantly, we show that the LRCS model can demonstrate an interesting negative correlation between the *B*- and *H*-values, which has been frequently implied in observations of seismicity and firstly verified in our present simulations. **Citation:** Lee, Y.-T., C.-C. Chen, T. Hasumi, and H.-L. Hsu (2009), Precursory phenomena associated with large avalanches in the long-range connective sandpile model II: An implication to the relation between the *b*-value and the Hurst exponent in seismicity, *Geophys. Res. Lett.*, *36*, L02308, doi:10.1029/2008GL036548.

1. Introduction

[2] In the estimation of seismicity tendency, the Gutenberg-Richter *b*-value and the Hurst exponent are two parameters which are widely used. Many literatures considered the *b*-value as a monitoring index related to the forthcoming large earthquakes [*Smith*, 1986; *Urbancic et al.*, 1992; *Wiemer and Wyss*, 1994; *Henderson et al.*, 1994; *Legrand et al.*, 1996; *Henderson et al.*, 1999; *Wyss et al.*, 2004; *Wu and Chiao*, 2006]. Reductions in the *b*-values before large earthquakes have been reported in many researches. The reduced *b*-value is probably caused by the quiescence of smaller earthquakes and/or the activation of moderate earthquakes [*Chen*, 2003; *Chen et al.*, 2005; *Wu and Chiao*, 2006]. Another parameter, the Hurst exponent, is based on the rescaled range (R/S) analysis, which was proposed by a hydrologist H. E. Hurst [*Hurst*, 1951]. The R/S analysis can figure out the statistical properties of time series and has also been used to analyze time series of earthquakes as the attempt on predicting future earthquake trends. Many groups of researchers have applied the R/S analysis to investigate the long-term correlation of seismicity [*Cisternas et al.*, 2004; *Goltz*, 1997; *Lomnitz*, 1994; *Telesca et al.*, 2001; *Chen et al.*, 2008c]. Each research group constructed their own time series for the R/S analysis from the earthquake catalogues. For example, *Cisternas et al.* [2004] had constructed the cumulative seismic moment as a function of time for conducting the R/S analysis of the seismicity in the Marmara Sea Region, Turkey. They showed the time variation of seismicity is persistent with the Hurst exponent *H* of 0.82. Also, *Telesca et al.* [2001] had analyzed the temporal fluctuations in *H* for the waiting time of earthquakes occurred in southern Italy and found the values of *H* range from 0.5 to 0.92. They also found a good correlation, with a correlation coefficient about -0.64, between the spectral power-law exponent of geo-electrical signals and the Hurst exponent of seismicity in Italy.

[3] In the present work, we calculate the Hurst exponent *H* and the power-law exponent *B* of the frequency-size distributions of avalanches in a modified sandpile model, the long-range connective sandpile (LRCS) model [*Chen et al.*, 2008a, 2008b; *Lee et al.*, 2008]. The upper-case *B* is exclusively used for the scaling exponent of the power-law frequency-size distribution of avalanches for differencing from the low-case *b* in the Gutenberg-Richter relation [*Gutenberg and Richter*, 1949]. The LRCS model differs from the original sandpile (BTW) model in the aspect of releasing grains to nearest neighboring cells. The LRCS model can release storing energy to remote cells, which is reminiscent of seismic wave propagations [*Chen et al.*, 2008a, 2008b; *Lee et al.*, 2008]. We show the increase in *H* values accompanied with the decrease in *B* values prior to large avalanches, which mimics the observed precursory phenomena of the Gutenberg-Richter *b*-values in real seismicity. Most importantly, we present the negative correlation between *B* and *H* in the LRCS model.

2. Long-Range Connective Sandpile (LRCS) Models

[4] *Bak et al.* [1987] proposed a concept of self-oragnized criticality (SOC) using a sandpile model, denoted here by the BTW model, and showed that the BTW model reaches a critical state without the need to fine-tune system parameters. Since then, the study of SOC has been investigated by simulating the BTW model and many modified versions of the BTW model. We have previously proposed a long-range connective sandpile (LRCS) model by introducing randomly remote connections between two separated cells [*Chen et al.*, 2008a, 2008b; *Lee et al.*, 2008]. The simulation was performed in the "stop-and-go" mode. When accumulated grains at one cell reached the threshold amount of 4 the redistribution process occurred. One of the original nearest-neighbor connections faces a connective probability P_c of redirecting to a randomly chosen, distant cell.

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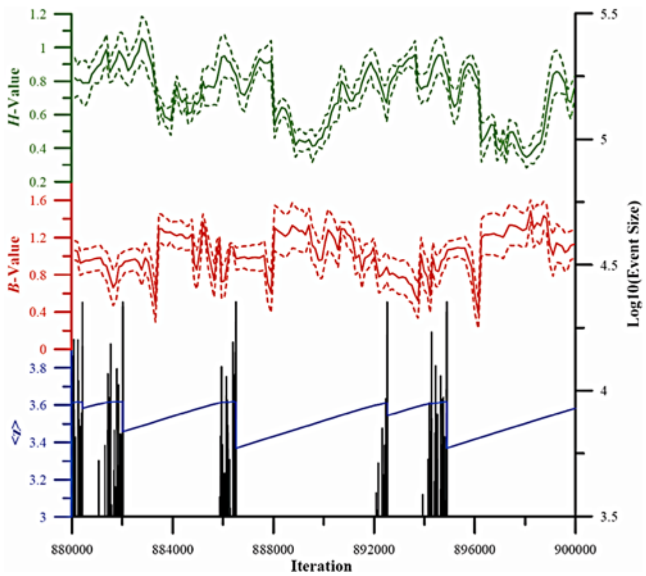

Figure 1. For a square grid of 150 by 150 cells, blue line represents the dynamic variable of the average topographic height of the sandpile, $\langle Z\rangle(t)$, in the LRCS model with self-adapted P_c. Green and red lines are the Hurst exponent and the power-law exponent of frequency-size distribution, respectively. Dash lines show their 95% confidence intervals. Also shown are occurrence times of avalanches (black bars) with sizes larger than $10^{3.5}$.

The original nearest-neighbor connection was thus replaced by the randomly chosen mesh which might be far from the toppling cell.

[5] The LRCS model differs from the BTW model in view of releasing grains to four nearest neighboring cells. By using a self-adapted probability threshold P_c of remote connection, the LRCS model can demonstrate the *intermittent criticality*, in which the sandpile approaches and retreats quasi-periodically from the critical state [*Chen et al.*, 2008b]. We assumed that P_c depends strongly on topographic change induced by the latest event [*Chen et al.*, 2008b; *Lee et al.*, 2008], which had simply been defined as $P_c\ (i+1) = [\Delta Z(i)/\alpha L^2]^3$ ($\alpha \cong 1.25$). $\Delta Z(i)$ and L^2 are topographic change due to the latest event and the system size, respectively. There may exist many different ways to exactly define P_c for different physical systems. However, to the end of demonstrating the intermittent criticality in the sandpile model, we found that the exact choice of formulation for P_c is not crucial [*Chen et al.*, 2008b]. The LRCS system after a large avalanche can thus evoke a higher connective probability P_c value, motivated by that a more active system will have higher probability to establish long-range connection due to fault activity, the change of pore water pressure or dynamic triggering of seismic waves. For example, a larger earthquake generates more radiated energy carried by seismic waves, thus, is more capable of dynamically triggering remote tremors far away the main shock. In those remotely triggering cases, stress perturbation due to seismic waves is considered as the immediate cause of triggered events.

[6] For the LRCS model with self-adapted P_c, the dynamic variable of the spatially averaged amount of grains on board $\langle Z\rangle(t)$ (=$(\sum_{i=1}^{L^2} Z_i)/L^2$, blue line in Figure 1) is often punctuated towards a smaller value by a large event. The large fluctuation in $\langle Z\rangle(t)$ is an important feature mimicking the intermittent criticality [*Sammis and Smith*, 1999; *Rundle et al.*, 2000; *Castellaro and Mulargia*, 2002; *Main and Al-Kindy*, 2002; *Goltz and Böse*, 2002]. In the LRCS model, large avalanches are then followed by a period of quiescence and a new approach back toward the critical state (Figure 1). Such process is similar to the dynamical process of the earthquake fault system which repeats by reloading energy and rebuilding correlation lengths towards criticality and the next great event [*Rundle et al.*, 2000]. For more details about the LRCS model, we refer the readers to our previous papers [*Chen et al.*, 2008a, 2008b; *Lee et al.*, 2008].

3. Calculations of *B*- and *H*-Values for Avalanches in the LRCS Model

[7] A numerical sandpile model with 10^6 throws of single grain on a square grid of 150 by 150 meshes has about 375,000 avalanche events. We calculated the power-law exponent B of the frequency-size distributions of avalanche events and the Hurst exponent H of avalanche sizes using every 500 events. The avalanche size was defined as the total number of cells which have reached the grain threshold of 4 and toppled during a complete avalanche. To trace variations in B and H with respect to time the sliding window technique with an overlap of 450 events is used, which means that we selected 500 events to calculate Bs and Hs then shifted 50 events to calculate the next values of B and H. For the calculation of B, we applied the data binning technique proposed by *Christensen and Moloney* [2005] to reduce the noise effect of large avalanches and then adopted the weighted least-square regression to fit the frequency-size distribution. As for the calculation of H, a brief summary of the R/S analysis is given below. The R/S analysis utilizes two factors: one is the range R, which is the difference between maximum and minimum amounts of accumulated departure of time series from the mean over a time span τ, and the other is the standard deviation S over the time span. The so-called rescaled range is exactly the ratio of R and S, i.e., R/S. Analyzing a variety of time series of natural phenomena, the toppling size of avalanche for example, it has been concluded that the ratio R/S is very well described by the empirical relation $(R/S)(\tau) = (\tau/2)^H$, where H is the Hurst exponent. For the independent random process, with no correlations among samples, $H = 0.5$. The observational time series is persistent for $H > 0.5$ whereas the sequence shows the anti-persistent behavior for $H < 0.5$. The concepts of persistent and anti-persistent memories in time are well defined for non-linear processes [*Feder*, 1988].

[8] Figure 1 shows the B-value variation (red line) and the Hurst exponent H variation (green line) with different time windows obtained from the LRCS model. Error bars show the 95% confidence intervals. As an example, Figure 1 displays the result of $2\cdot10^4$ iterations among 10^6 from the LRCS model, showing five sequences of system-wide avalanches with sizes close to L^2 (=22500). B ranges from 0.3 to 1.6 and H from 0.3 to ~1. We had found, in the LRCS models with smaller meshes of 50 by 50 and 100 by 100, the power-law exponents (the B values) derived from the frequency-size distributions of avalanches decrease before large events [*Lee et al.*, 2008], which is reminiscent of pre-

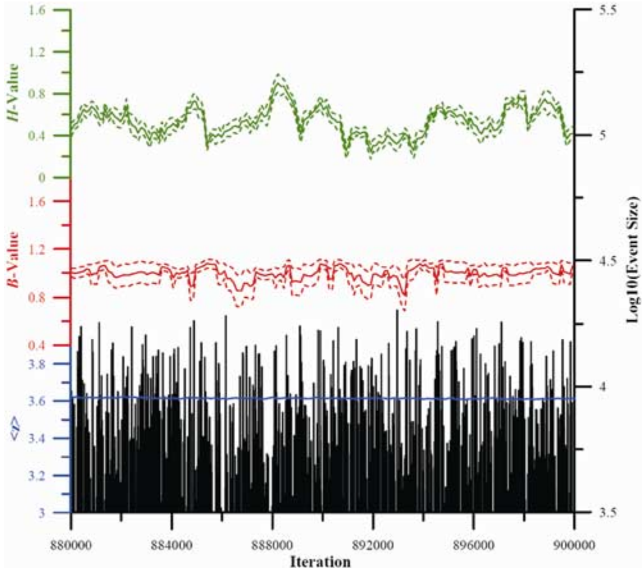

Figure 2. For a square grid of 150 by 150 cells, blue line represents the dynamic variable of the average topographic height of the sandpile, $\langle Z \rangle(t)$, in the BTW model. Green and red lines are the Hurst exponent and the power-law exponent of frequency-size distribution, respectively. Dash lines show their 95% confidence intervals. Also shown are occurrence times of avalanches (black bars) with sizes larger than $10^{3.5}$.

cursory Gutenberg-Richter's *b*-value reduction before large earthquakes [*Smith*, 1986; *Urbancic et al.*, 1992; *Wiemer and Wyss*, 1994; *Henderson et al.*, 1994; *Legrand et al.*, 1996]. Here, by using the present larger grid size of sandpiles, we again confirm the precursory declines in the *B*-values. On the other hand, importantly, we find that the change of *H* has fluctuation in the opposite sense to the variation in *B*. While the *B* usually increases following a large avalanche, the *H* usually reduces. The *H*-value then increases prior to the next large avalanche. Comparing *B* and *H* in Figure 1, we can find an interestingly negative correlation between *B* and *H*. The fact that *B*-values typically reduce and *H*-values increase prior to large avalanches reinforces the view that there is possibility of detecting precursors of great events in the LRCS model [*Lee et al.*, 2008]. A simple statistical counting shows that about 90% of those system-wide events exhibit precursory phenomena of reduction in *B* and increase in *H*. In Figure 2, for comparison, we present results obtained from the original BTW model. Neither precursory phenomena prior to large avalanches nor negative correlations between *B* and *H* could be found in the original BTW sandpile model with a square grid of 150 by 150 meshes. The values of *H* calculated from the BTW model are almost near 0.5, indicating a lack of memory effect, together with the *B*-values around 1.

[9] To make the relation between *B* and *H* clearer, we plot the points of (*B*, *H*) in a scatter graph showing the correlation between these two parameters (Figure 3). Blue circles and red crosses in Figure 3 represent results obtained from the LRCS and BTW models, respectively. It is true that the points of (*B*, *H*) from the LRCS model (blue circles) are somewhat distributed messily and the statistical correlation coefficient is only about −0.52. Statistically the averaged *H* dependence on *B* (black squares in Figure 3) obtained from a set of *B*-bins with a step of 0.1 could help visualize the negative correlation between them. Nevertheless, when comparing with the result from the BTW model (red crosses), they demonstrate a good dependence of these two parameters upon each other and indicate the strikingly negative correlation between *B* and *H* in the LRCS model.

4. Conclusion and Discussions

[10] The LRCS model considering randomly remote connections demonstrates the state of the intermittent criticality, where a quasi-periodic behavior of approaching and retreating from the criticality exists. In the present paper, we show that both the power-law exponent *B* and the Hurst exponent *H* are relevant to detect precursory phenomena prior to a forthcoming large event. Note that as the *B*-value starts to decrease the *H*-value starts to increase, indicating that the system starts to prepare for the next large event. The dynamic variable $\langle Z \rangle(t)$, i.e., the averaged height, in the LRCS model moving up to the critical state seemingly experiences the occurrence of a large avalanche. Such precursory phenomena were frequently reported in real seismicity.

[11] The decreasing *B* indicates that the number of moderate to large avalanches increases as the system approaches the critical state. The increase of the *H*-value can be interpreted as an increase of the degree of the persistence of larger and larger avalanches in the LRCS model. The long-range connective probability P_c undoubtedly plays a crucial role to the precursory phenomena in the LRCS model. When the long-range connection happens to the system, it facilitates an initiation of another local avalanche in the other end of the remote connection and nucleates new local avalanche which thus *averagely* increases the avalanche activities [*Lahtinen et al.*, 2005]. Since the P_c in our LRCS model

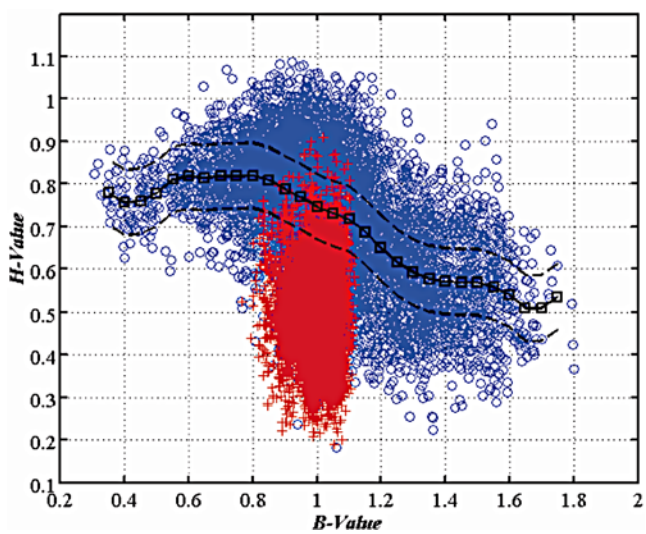

Figure 3. Scatter plot of the Hurst exponent *H* and the power-law exponent *B* of frequency-size distribution obtained from the avalanche events of the LRCS (blue circles) and BTW (red crosses) models. Black squares are the average *H*(*B*) dependence obtained from a set of *B*-bins with a step of 0.1. Black dotted lines show the corresponding standard deviations.

depends on the latest event size, it thus gives rise to a positive feedback in the system. Large event induces high probability releasing energy to far sites, which furthermore causes a chain of local reactions in toppling around the sandpile model. Plausibly the large-scale correlation in the spatial distribution of energy gradually grows [*Uritsky et al.*, 2004], which consequently produces a series of larger and larger events.

[12] Also similar to real seismicity, while the BTW model has a high percentage of more than 65% for the H-value around 0.5, the LRCS model has more than 70% of the Hs larger than 0.6. *Chen et al.* [2008c] have reported that the Hurst exponent of slip data in real earthquake catalogue usually possesses a value greater than 0.6, showing the persistent memory effect. It, however, should be noticed that the estimated H values are probably biased for sandpile models with different grid sizes since finite grid sizes may change the fractal properties of the investigated time series.

[13] As an important result, the changes in the opposite sense for the B- and H-values have been found in the intermittent critical system of the LRCS model. We have furthermore simulated a larger size of square grid with 250 by 250 meshes and found the similar negative correlation between B and H. Discussion on the origin of the negative correlation between the B- and H-values of earthquake time series is fundamental. On the basis of the fractional Brownian motion (fBm), *Voss* [1989] presented the correlation between the H and the spectral density exponent β, i.e., $\beta = 2H + 1$. By transforming spectral densities $S(f)$ and frequency f to seismic moment E and number of events $N(E)$, respectively, *Wu* [2001] yielded the power-law frequency-size distribution $N(E) \sim E^{-B}$ and the relation of $B = 1/\beta$. Consequently an inverse correlation, looked like the negative correlation, between H and B could be obtained. On the other hand, *Frankel* [1991] developed a model of complex self-similar rupturing where an earthquake is composed of sub-events with different sizes. By assuming that sub-events on a fault were proportional to fault strength, he showed the fractal dimension D of sub-events is controlled by a fault strength, which is scaled by a distance along the fault. By analyzing stress along a fault plane and comparing the correlation between the number of sub-events and fault radius, he furthermore derived the correlation between Gutenberg-Richter's b-value and D: $b = 1.5D/(3 + H)$, where the Hurst exponent H describes the scaling of stress drop with source radius. Since $B \sim b$, again, an inverse correlation between H and B could be obtained.

[14] Also interestingly, *Telesca et al.* [2001] proposed a new approach to investigate the correlation between electrical signals and earthquakes. They found a good correlation between the spectral power-law exponent α of geo-electrical signals and the Hurst exponent H of seismicity in their studied area. The correlation coefficient of α and H is about -0.64 in their study. In the present study we revealed a similar negative correlation between the Hurst exponent of avalanches and the power-law slope of frequency-size distributions of events. Whether there exists any connection between these studies remains an interesting debate.

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References

Bak, P., C. Tang, and K. Wiesenfeld (1987), Self-organized criticality: An explanation of 1/*f* noise, *Phys. Rev. Lett.*, *59*, 381–384.
Castellaro, S., and F. Mulargia (2002), What criticality in cellular automata models of earthquakes?, *Geophys. J. Int.*, *150*, 483–493.
Chen, C.-C. (2003), Accelerating seismicity of moderate-sized earthquakes before the 1999 Chi-Chi, Taiwan, earthquake: Testing time-prediction of self-organizing spinodal model of earthquakes, *Geophys. J. Int.*, *155*, F1–F5.
Chen, C.-C., J. B. Rundle, J. R. Holliday, K. Z. Nanjo, D. L. Turcotte, S.-C. Li, and K. F. Tiampo (2005), The 1999 Chi-Chi, Taiwan, earthquake as a typical example of seismic activation and quiescence, *Geophys. Res. Lett.*, *32*, L22315, doi:10.1029/2005GL023991.
Chen, C. C., L. Y. Chiao, Y.-T. Lee, H. W. Cheng, and Y. M. Wu (2008a), Long-range connective sandpile models and its implication to seismicity evolution, *Tectonophysics*, *454*, 104–107.
Chen, C.-C., Y.-T. Lee, and L. Y. Chiao (2008b), Intermittent criticality in the long-range connective sandpile (LRCS) model, *Phys. Lett. A*, *372*, 4340–4343.
Chen, C.-C., Y.-T. Lee, and Y. F. Chang (2008c), A relationship between Hurst exponents of slip and waiting time data of earthquakes, *Physica A*, *387*, 4643–4648.
Christensen, K., and N. R. Moloney (2005), *Complexity and Criticality*, *Imperial Coll. Press Adv. Phys. Texts*, vol. 1, Imperial Coll. Press, London.
Cisternas, A., O. Polat, and L. Rivera (2004), The Marmara Sea region: Seismic behaviour in time and the likelihood of another large earthquake near Istanbul (Turkey), *J. Seismol.*, *8*, 427–437.
Feder, J. (1988), *Fractals*, Plenum, New York.
Frankel, A. (1991), High-frequency spectral falloff of earthquake, fractal dimension of complex rupture, *b* value, and the scaling of strength on faults, *J. Geophys. Res.*, *96*, 6291–6302.
Goltz, C. (1997), *Fractal and Chaotic Properties of the Earthquakes*, *Lecture Notes Earth Sci.*, vol. 77, Springer, Berlin.
Goltz, C., and M. Böse (2002), Configurational entropy of critical earthquake populations, *Geophys. Res. Lett.*, *29*(20), 1990, doi:10.1029/2002GL015540.
Gutenberg, B., and C. F. Richter (1949), *Seismicity of the Earth and Associated Phenomena*, Princeton Univ. Press, Princeton, N. J.
Henderson, J., I. G. Main, R. G. Pearce, and M. Takeya (1994), Seismicity in north-eastern Brazil: Fractal clustering and the evolution of the *b* value, *Geophys. J. Int.*, *116*, 217–226.
Henderson, J. R., D. J. Barton, and G. R. Foulger (1999), Fractal clustering of induced seismicity in the Geysers geothermal area, California, *Geophys. J. Int.*, *139*, 317–324.
Hurst, H. E. (1951), Long-term storage capacity of reservoirs, *Am. Soc. Civil Eng. Trans.*, *2447*, 770–808.
Lahtinen, J., J. Kertesz, and K. Kaski (2005), Sandpiles on Watts-Strogatz type small-worlds, *Physica A*, *349*, 535–547.
Lee, Y. T., C.-C. Chen, L. Y. Chiao, and Y. F. Chang (2008), Precursory phenomena associated with large avalanches in long-range connective sandpile (LRCS) model, *Physica A*, *387*, 5263–5270.
Legrand, D., A. Cisternas, and L. Dorbath (1996), Multifractal Analysis of the 1992 Erzincan Aftershock Sequence, *Geophys. Res. Lett.*, *23*, 933–936.
Lomnitz, C. (1994), *Fundamentals of Earthquake Prediction*, John Wiley, New York.
Main, I. G., and F. H. Al-Kindy (2002), Entropy, energy, and proximity to criticality in global earthquake populations, *Geophys. Res. Lett.*, *29*(7), 1121, doi:10.1029/2001GL014078.
Rundle, J. B., W. Klein, D. L. Turcotte, and B. D. Malamud (2000), Precursory seismic activation and critical-point phenomena, *Pure Appl. Geophys.*, *157*, 2165–2182.
Sammis, C. G., and S. W. Smith (1999), Seismic cycles and the evolution of stress correlation in cellular automaton models of finite fault networks, *Pure Appl. Geophys.*, *155*, 307–334.
Smith, W. D. (1986), Evidence of precursory changes in the frequency-magnitude *b* value, *Geophys. J. R. Astron. Soc.*, *86*, 815–838.
Telesca, L., V. Cuomo, and V. Lapenna (2001), A new approach to investigate the correlation between geoelectrical time fluctuations and earthquake in a seismic area of southern Italy, *Geophys. Res. Lett.*, *28*, 4375–4378.
Urbancic, T. I., C. I. Trifu, J. M. Long, and R. P. Young (1992), Space-time correlations of *b* values with stress release, *Pure Appl. Geophys.*, *139*, 449–462.

Uritsky, V., N. Smirnova, V. Troyan, and F. Vallianatos (2004), Critical dynamics of fractal fault systems and its role in the generaion of pre-seismic electromagnetic emissions, *Phys. Chem. Earth*, *29*, 473–480.

Voss, R. F. (1989), Random fractals: Self-affinity in noise, music, mountain, and clouds, *Physica D*, *38*, 362–371.

Wiemer, S., and M. Wyss (1994), Seismic quiescence before the landers (M = 7.5) and big bear (M = 6.5) 1992 earthquakes, *Bull. Seismol. Soc. Am.*, *84*, 900–916.

Wu, Y. M., and L. Y. Chiao (2006), Seismic quiescence before the 1999 Chi-Chi, Taiwan Mw7.6 earthquake, *Bull. Seismol. Soc. Am.*, *96*, 321–327.

Wu, Z. L. (2001), A discussion on the application of *b*-value to the prediction of seismic tendency, *Acta Seismol. Sin.*, *14*, 585–588.

Wyss, M., C. G. Sammis, R. M. Nadeau, and S. Wiemer (2004), Fractal dimension and *b*-value on creeping and locked patches of the San-Andreas fault near Parkfield, California, *Bull. Seismol. Soc. Am.*, *94*, 410–421.

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