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A relationship between Hurst exponents of slip and waiting time data of earthquakes

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ABSTRACT

We propose a new relationship between the slip and waiting time of real earthquake series. We calculated the Hurst exponents for both time series of slip and waiting time of earthquake sequence in Taiwan CWB (Central Weather Bureau) catalogue. Our findings suggest a good correlation with a correlation coefficient of about 0.8 between the two exponents. Such a good correlation is highly similar to the ones expected from time- or slip-predictable earthquake recurrence models and suggests that the recurrence of real seismicity could be reduced to the time- or slip-predictability in certain sense. This paper, thus, initiates a new direction re-considering earthquake recurrence.

1. Introduction

Two end-member models for earthquake recurrence are the time- and slip-predictable models [16,10]. In the time-predictable model an earthquake always happens when the stress level reaches the threshold of the material's strength. The slip during the earthquake is randomly variable, as well as the dropping amount of the cumulate stress. Once the slip in an event has been measured, then it could be known when the next event will occur, providing the cumulative rate of the stress from plate motion is fixed. On the other hand, in the slip-predictable model, the stress always drops to the same lower level after an earthquake. However, there is no fixed upper threshold of the stress to be reached for every event. Also the occurrence time of an event is randomly variable. The longer the time since the last earthquake, the larger the next event will be in the context of slip-predictable model. Most importantly, there is only one degree of freedom in both models: for the former that is the slip of an earthquake and for the later the inter-event time between two successive events (i.e. the waiting time of the next earthquake). Unfortunately, from the standpoint of hazard prevention, representing the real seismicity with either the time- or slip-predictable model had often been reported as infeasible in the published literatures (e.g. Refs. [19,13,14]). Those negative facts suggest that the slip and the waiting time for real seismicity could probably be independent of each other. In contrast, we in this paper will present a new kind of relationship, based on Hurst's rescaled range analysis [8,4], between the slip and waiting time of real earthquake series.

2. Hurst's rescaled range analysis

The rescaled range statistical analysis (R/S analysis, for short) was originally developed by a hydrologist Hurst [8] and is now commonly used as a method to detect correlations in time series of physical quantity for natural phenomena, after the

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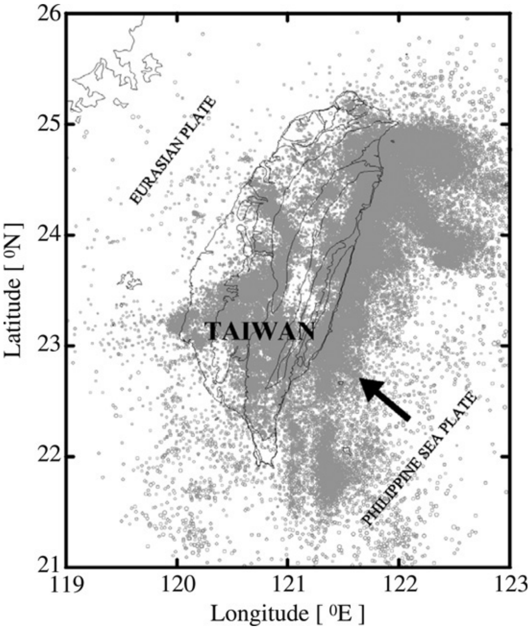

Fig. 1. Map showing the epicenters of earthquakes used in this study. Thick arrow indicates the moving direction of relative motion between the Eurasian Plate and the Philippine Sea Plate.

works of Hurst et al. [7], Mandelbrot and Wallis [11,12] and Feder [4]. Also, Ref. [5] provides a systematic demonstration which shows the R/S analysis is a very sensitive method to reveal hidden long-run and short-run correlations in the pseudorandom number records. A brief summary for the R/S analysis is given below.

There are two factors used in the R/S analysis: first, the range R, which is the difference between the maximum and minimum amounts of accumulated departure of the time series from the mean over a time span τ, and second, the standard deviation S calculated over the time span τ. The so-called rescaled range is exactly the ratio of R and S. From a variety of time series of natural phenomena, it is concluded that the ratio R/S is very well described by the following empirical relation:

$$R/S = (\tau/2)^H,$$

where H is the Hurst exponent. The Hurst exponent is also related to the fractal dimension D. If the time series is self-similarity and self-affine, the relationship between those two parameters could be expressed as $D = 2 - H$ [4].

This statistics handles the evolution of observations in time. The correlation of the past and the future in the observational time series can be described by the H. For independent random process, with no correlations among samples, $H = 0.5$. For $H > 0.5$, the observational time series is of persistence, which means averagely the increasing (decreasing) trend in the past induces the continued increasing (decreasing) trend in the future. On the other hand, when $H < 0.5$ the sequence is characterized by the anti-persistent behavior, meaning that an increasing (decreasing) trend in the past causes a decreasing (increasing) trend in the future. The concepts of persistent and anti-persistent memories in time are well defined for non-linear processes [4].

3. Analysis of Taiwan CWB earthquake catalogues

The earthquake catalogue (Fig. 1) used in this study is released from the CWB of Taiwan [17] and includes data of earthquakes occurring in the Taiwan area from 1991 through 2000. CWB began to install a new local seismic network, CWBSN, at the beginning of 1990s. For high performance seismic monitoring, digital three-component short-period velocity sensors and force balanced accelerometers were installed at all 73 stations. The catalogue released in Ref. [17] is the most definitive record produced by modern seismic monitoring work in Taiwan. Shown in Fig. 2 is the frequency-magnitude distribution of earthquakes in the CWB catalogue. The magnitude distribution of earthquakes larger than 2, could be able to fulfill the Gutenberg–Richter law very well, with the b value of $\sim$0.8 (Fig. 2). Therefore, for the requirement of a complete catalogue, we consider magnitude 2 as the completeness magnitude throughout this study.

To calculate the slip data from the earthquake catalogue without the details of rupture dynamics, a simple transformation has been used [9]. With the use of elastic dislocation theory, a small fault with the area A and displacement offset D can be represented by a force double couple with the moment of each force couple, $M_0 = \mu DA$, where μ is the rigidity of the medium surrounding the source. Statistically, the scaling relation between seismic moment and the fault area is usually found to be $M_0 \sim A^{3/2}$ (e.g. Fig. 11 of Ref. [9]). Therefore, we consider the average slip $< D >$ of an event is simply proportional to the cubic root of the seismic moment, which could be directly converted from the local magnitude [20] in the CWB catalogue.

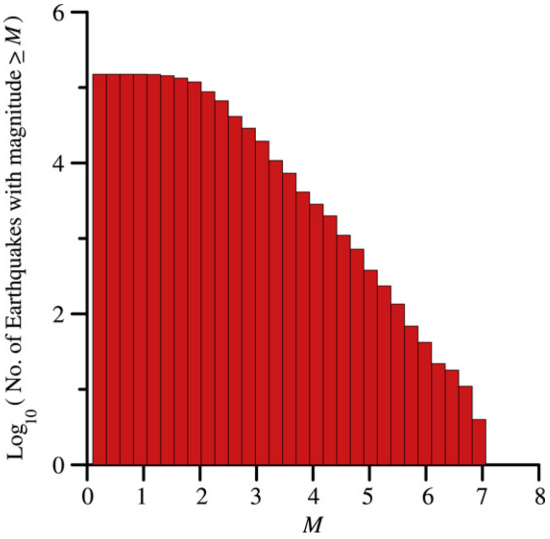

Fig. 2. Frequency-magnitude distribution of earthquakes in the CWB catalogue.

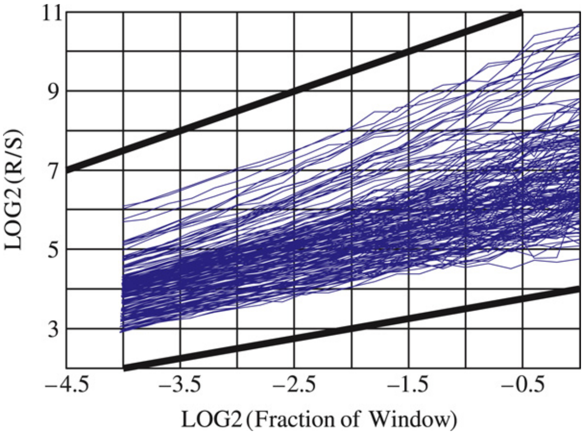

Fig. 3. Examples of calculated R/S curves for many data sets, each with 2000 h. The Hurst exponent was estimated from the slope in the double logarithm plot. The timescale range, over which the Hurst exponent was estimated, can extend from about 100–2000 h. Upper and lower bold lines represent the slopes of 1.0 and 0.5, respectively.

The evolution of H with respect to time has been carried out with a fixed time window of 2000 h and a shift of 500 h between successive windows, in order to have a sufficient amount of data points to accurately and smoothly estimate the scaling exponent H. The sliding process applied to the calculation of two kinds of H for both the slip and waiting time data. Averagely speaking, there are about 2000 earthquakes with magnitude larger than 2 occurred within a time window of 2000 h. Fig. 3 shows examples of our R/S curves (Hurst plots) for many data sets, from which the two kinds of Hurst exponents were estimated. The timescale range, over which the empirical power-law relation can fairly fit the R/S values, extends from about 100–2000 h.

Fig. 4 shows a scatter plot of such two kinds of H from the slip and waiting time data of the CWB earthquake catalogue. Surprisingly, there exists a good dependence upon each other. The statistical correlation coefficient for these two kinds of H is about 0.8. For comparison, also shown in Fig. 4 is the result of a synthetic earthquake catalogue obtained from the time-predictable model, and its correlation coefficient is above 0.95. To synthesize the catalogue, we first produced a set of earthquake magnitudes, arranging randomly but obeying the Gutenberg–Richter distribution. Then, strains had accumulated in a constant rate. When cumulative strain reached a prescribed upper threshold, an artificial earthquake occurred with a slip transformed from the aforementioned magnitude dataset. As mentioned before, because the waiting time of the next earthquake is purely a function of the slip induced by last earthquake, the high correlation coefficient between the two Hurst exponents can be totally expected. However, one significant difference between the real and synthetic catalogue is the coverage of H for the real catalogue, which extends from $\sim$0.4 to $\sim$1, is larger than the synthetic one. It strongly suggests that the so-called "memory" effect may exist in the real earthquake data [3,2].

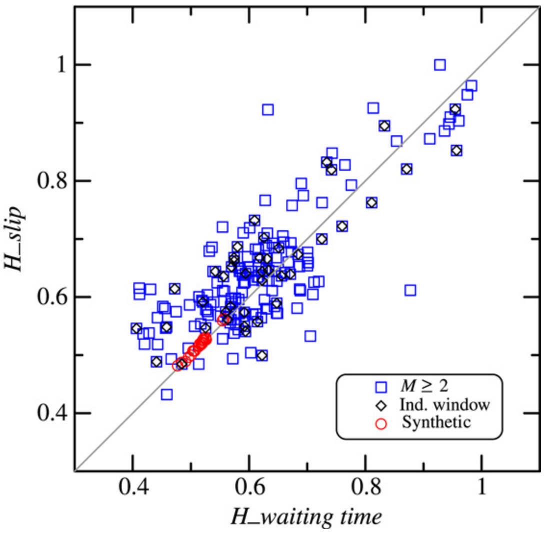

Fig. 4. Scatter plot of the Hurst exponent H calculated from the slip and waiting time data of the CWB earthquake catalogue (squares with a statistical correlation coefficient of 0.8 for overlapping time windows; diamonds with the correlation coefficient of 0.85 for independent, non-overlapping windows), also from a synthetic catalogue based on the time-predictable earthquake recurrence model (circles). While the strong correlation between the two Hs is expected for the synthetic catalogue of time-predictable model, the good correlation for real catalogue indicates a hidden dynamical behavior for the earthquake fault system, which could provide a new direction for reconsidering the earthquake recurrence.

We also analyzed the time series of earthquake data with different magnitude cutoffs, M_c. As shown in Fig. 5 are the results for $M_c = 2$ (squares), 2.5 (crosses) and 3 (circles). Obviously many of the Hurst exponents for the waiting time data with $M_c = 3$ biased towards lower values than those Hs with $M_c = 2$ or 2.5, while the magnitude cutoff had a little effect upon the Hurst exponents for the slip data. Such a result seems understandable to us. When calculating the Hurst exponent, the range and standard deviation of slip data are mainly controlled by larger earthquakes. Consequently, although the dataset with a smaller M_c (e.g. 2 or 2.5) includes many small events, they do not induce the Hurst exponent to change so much. On the contrary, a large time span could be broken into many small spans by many inserts of small events due to a smaller M_c. The underlying structure of waiting time data therefore could be severely changed. In other words, a dataset with larger M_c loses more information about the underlying dynamics of earthquakes than with smaller M_c, and therefore represents another type of “incomplete” data.

To further confirm the correlation between the Hs of the slip and waiting time data in real data, we reshuffle the CWB catalogue by means of randomly permuting the order of the slip and waiting time sequences, and re-calculate the Hs. The strong correlation disappeared, indeed, after reshuffling the real earthquake catalogue (Fig. 6). Meanwhile the shrink of the coverage of the Hs indicates the absence of the “memory” effect.

4. Concluding remarks and discussion

Since 1990s many groups of researchers have applied the R/S analysis to investigate the long-term correlation of seismicity (e.g. Refs. [15,10,3,6,18,2]. Each research group constructed their own time series for the R/S analysis from the earthquake catalogue. Telesca et al. [18], for example, had analyzed the temporal fluctuations of the Hurst exponent H for the waiting time of the earthquakes occurred in southern Italy and found the values of H range from 0.5 to 0.92 with a mean of 0.74. Cisternas et al. [2] had constructed the cumulative seismic moment as a function of time for conducting the R/S analysis of the seismicity in the Marmara Sea Region, Turkey, and concluded that the aftershocks of the 1999 Izmit earthquake show an exceptionally high persistent memory with the H of 0.95.

To our knowledge, the present work is the first work that *simultaneously* calculates the H for the slip and waiting time records of the earthquake system. Most importantly, comparison between the two Hurst exponents exposes a good correlation hidden in the stochastic processes of the slip and waiting time behaviors for the earthquake system of Taiwan. Two critical tasks need more investigations in the near future. Firstly, could such a correlation be present in other earthquake systems worldwide? Secondly, based on the standpoint of phase dynamics, is it possible to construct a rule connecting the slip and waiting time behavior of the earthquake system? These works may initiate a new direction for re-considering the earthquake recurrence model.

Finally, we emphasize that no spatial selection of earthquakes had been made for our R/S analysis here. The original motivation for doing this comes from the concept of critical earthquake. In the context of critical earthquake, two *related* earthquakes may be separated from each other by much longer distance than expected conventionally [1]. It is also interesting to examine the size effect of study area chosen. The result of the Hs calculated from a smaller area, bounded within 120°E–122°E, 22°N–25°N and depth of 40 km, is also shown in Fig. 6. We found that, for the earthquake data in

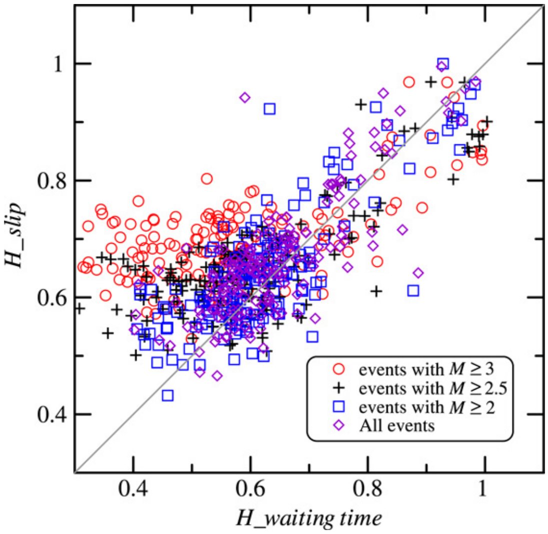

Fig. 5. Scatter plot of the Hurst exponent *H* calculated from the slip and waiting time data of the CWB earthquake catalogue with cutoffs at different magnitudes, M_c. There exists a significant correlation at $M_c = 2$ (squares) that becomes distorted at $M_c = 3$ (circles). For details please see the text.

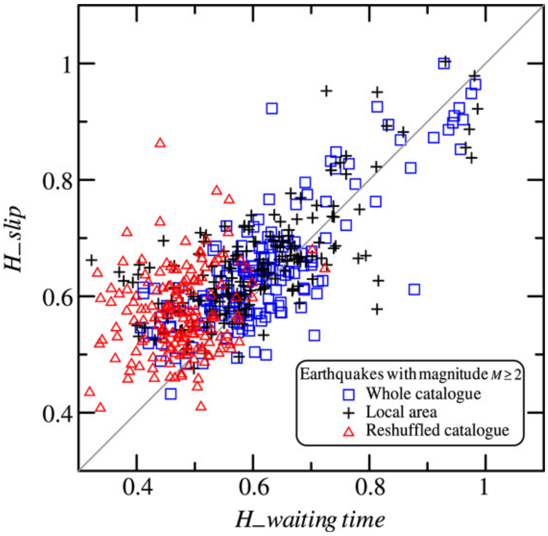

Fig. 6. Scatter plot of the *H*s calculated from the reshuffled catalogue (triangles) and a subset of the catalogue within a smaller spatial area (crosses). The symbol of square is the same as Fig. 4. For details please see the text.

this local area, the correlation coefficient between the two Hurst exponents reduces to 0.7 and several *H*s of waiting time data bias toward lower values (Fig. 6). It should be noticed that such additionally spatial restriction makes the structure of waiting time data changed severer than the slip data. We postpone to future work addressing the size effect and issues above-mentioned as well.

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